Mockexam algebraic topology November 2025

Always explain your answers. It is allowed to refer to definitions, lemmas and theorems from the lecture notes but not to other sources. All questions are independent and count equally so make sure you try each of them. Good luck!

1. In the course we classified finite connected surfaces without boundary up to stellar equivalence. What would happen if we tried to classify such surfaces up to simple homotopy equivalence? Would there be more equivalence classes, fewer equivalence classes or would it not make any difference? Discuss.

2. Write down matrices for ∂_2 and ∂_1 so as to compute an explicit basis for $H_1(\mathcal{A};\mathbb{F}_2)$ where

$$\mathcal{A} = \langle\{\{0,1,2\},\{3,4,5\},\{0,3\},\{1,4\},\{2,5\}\}\rangle$$

3. Define $J_n = \langle\{\{k,k+1\} : k \in \{0,1,\dots n\}\}\rangle$ and define S to be the set of all simplicial maps $f : J_n \to J_n$ with the property that $f(0) = 0$ and $f(n) = n$. How many elements does S have as a function of n? Prove your results.

4. Suppose we have an ASC $\mathcal{A}$ and an ASC $\mathcal{B}$ such that $H_n(\mathcal{A}\cup\mathcal{B};\mathbb{Q}) = 0$ for all $n > 3$. Prove that for $n > 5$ we have $\dim H_n(\mathcal{A}\cap\mathcal{B}) = \dim H_n(\mathcal{A};\mathbb{Q}) + \dim H_n(\mathcal{B};\mathbb{Q})$.

5. Given a connected surface Σ with precisely two boundary components. Prove that $\pi_1(\Sigma, \Delta^c_{\partial\Sigma}\Sigma, c)$ cannot be the trivial group.

6. Define an equivalence relation on $\mathcal{A} = (\mathbb{S}^2)'$ by $A \sim B$ iff $A \cup B = V(\mathbb{S}^2)$ and $A \cap B = \emptyset$. The quotient map $p : \mathcal{A} \to \mathcal{A}/\sim$ sending any vertex in $\mathcal{A}$ to its equivalence class is a covering map (you do not have to prove this). Find an element of the fundamental group of $\mathcal{A}/\sim$ with base point $\overline{\{0\}} \in \mathcal{A}/\sim$ that sends $\{0\}$ to $\{1,2,3\}$ under the monodromy action.

7. Suppose we have a sequence of ASC and simplicial maps $\mathcal{A}_n \xrightarrow{f_n} \mathcal{A}_{n-1} \xrightarrow{f_{n-1}} \dots \xrightarrow{f_1} \mathcal{A}_0$ such that $\{\dim \mathcal{A}_n\} = \{0,1,2,\dots n\}$. Prove that $g = f_1 \circ f_2 \circ \dots \circ f_n$ is such that $g_*(\alpha) = 1$ for any $\alpha \in \pi_1(\mathcal{A}_n, p)$ and $p \in V(\mathcal{A}_n)$.